Radiation Production Notes
Note 9

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Magnetic Pinching in Vacuum Diodes at Relativistic Electron Energies

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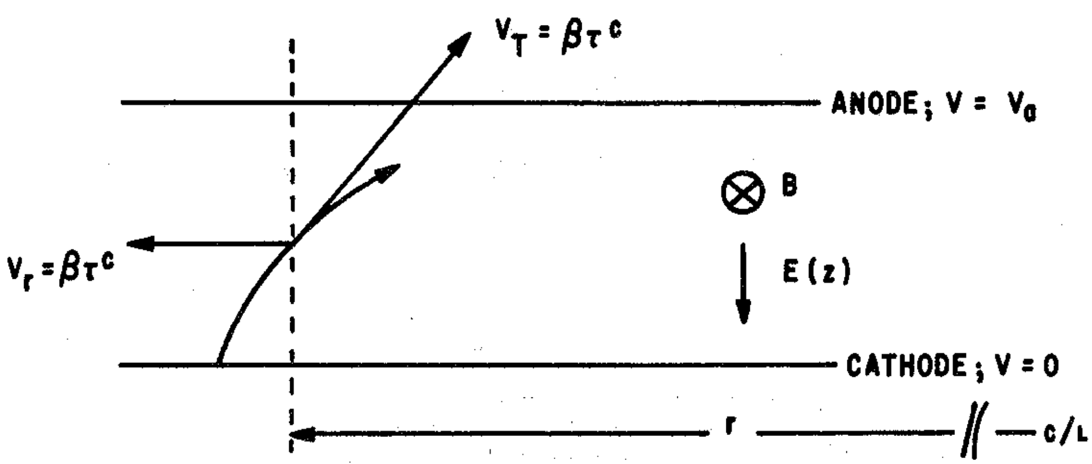
1. <u>The Pinch Criterion</u>

The net radial force on an accelerated electron in a plane parallel electrode system is determined by a positive electric field term and a negative magnetic field component. In a relatively low impedance diode however two effects combine to reduce the radial electric force component to negligible proportions. For ratios of electron 'beam' diameter to electrode separation much greater than unity the radial field which would be set up by the space charge density of the electrons is almost completely removed by the presence of the electodes - the field is virtually "shorted out". Secondly, and of less significance, the presence of ions in the gap originating from the background gas or, probably of more importance, from the electrodes themselves reduces the net negative charge density. The result is a net force directed radially inwards (in a cylindrically symmetric system) and determined solely by the magnetic field of the primary electron stream. The accelerated electron then describes a "relativistic cycloidal" orbit and at some magnetic field value just grazes the anode. We define this as the critical field and the associated ν/γ as the critical value.

Assume a parallel plane cylindrically symmetric system with uniform current density. Also (to keep the problem within the very limited bounds of my maths) assume that the aspect ratio r_C/s (cathode or outer electron radius to gap length) is large enough that the magnetic flux density B is roughly constant over the dimensions of the radial variation of a peripheral electron trajectory.

Then, for the system:



Orbit equations;

$$\frac{\partial}{\partial t}(mv_r) = -eBv_z \qquad \ldots\ldots \quad 1.$$

$$\frac{\partial}{\partial t}(mvz) = eE - eBv_r \qquad \ldots\ldots \quad 2.$$

Energy equation;

$$\left[1 - \frac{\nu_r^2 + \nu_3^2}{c^2}\right]^{-1/2} = 1 + \frac{V(z)}{V_o} \qquad \ldots\ldots \quad 3.$$

where $V_o = m_o c^2/e$

Integrating 1. gives:

$$|\beta_r| = \left|\frac{\nu_r}{c}\right| = \frac{eBz}{\gamma m_o c} \qquad \ldots\ldots \quad 4.$$

which expresses the radial velocity component as a function of z.

For a peripheral electron to just graze the anode, $\beta_z = 0$ and $\beta_r = \beta_T$. Then 3. gives:

$$|\beta_{ra}| = |\beta_T| = \frac{\sqrt{\gamma_a^2 - 1}}{\gamma_a} \qquad \ldots\ldots \quad 5.$$

γ_a being the value of γ at the anode plane, and 4. gives:

$$|\beta_{ra}| = \frac{eBs}{\gamma_a m_o c} \qquad \ldots\ldots \quad 6.$$

As $B = \mu_o I/2\pi r_c$ in the cylindrical system, r_c being the outer cathode radius, and ν/γ at the anode can be written;

$$\nu/\gamma_a = \frac{30I}{V_o} \quad \frac{1}{\sqrt{\gamma_a^2 - 1}} \qquad \ldots\ldots \quad 7.$$

then 5., 6., and 7. eventually give for the value of ν/γ_a which just allows the peripheral electron to reach the anode:

$$\left.\frac{\nu}{\gamma_a}\right|_{CRIT} = \frac{r_c}{2S} \qquad \ldots\ldots \quad 8.$$

-- the desired result.

Comments on 8.

As previously stated a constant magnetic field has been assumed. Even in the ideal uniform current density case this is not true - the field has a 1/r dependence. The accuracy of the result will depend on the ratio r_c/s and the radial variation of the electron trajectory. In the non-uniform case where emission takes place from discreet patches further corrections would be necessary. However, for an edge electron of an edge emitter, as long as the number of emitters is greater than about 10, the result will not be far wrong. Each individual beam in such an array has an associated individual critical value of ν/γ. For n uniformly distributed emitters producing equal currents and an emission area to total area ratio of α,

$$\frac{\nu/\gamma_{individual}}{\nu/\gamma_{ind.\ crit.}} \sim \frac{1}{\sqrt{n\alpha}} \qquad \ldots\ldots \quad 9.$$

so that as long as $n\alpha >> 1$, individual collapse will not be a problem. As $n\alpha \to 1$, electron orbits will be perturbed by the local field of individual emitters and the overall critical ν/γ modified. Relation 9. in theory is a criterion which helps to determine the emitter distribution. (For a given α, n must be chosen so that $n\alpha >> 1$.) In practice, however, for the sort of α of present interest (0.1 - 0.5), the condition that $n\alpha > 1$ is usually satisfied by determining n from considerations of overall beam uniformity.

2. Diode Gap Design

Two relationships relate to current flow in the diode gap:

a/ The modified Child-Langmuir space charge limited flow theory, which can be written in the form;

$$Z = \frac{ks^2}{r_e{}^2 V^{1/2}} \qquad \ldots\ldots \quad 10.$$

where r_e is the effective radius of the electron emission region ($\neq r_c$ in general), and k is an experimentally determined "constant" in the voltage range of interest -- a function of time, s, r_e and the phases of the moon, and which incorporates for our purposes the effect of s and r_e variations on Z.

b/ The Pinch criterion;

$$\frac{\nu}{\gamma} < \frac{r_c}{2s} \qquad \ldots\ldots \quad (8)$$

A third approximate criterion is derived from the experimental observation that catastrophic impedance collapse sets in during the time of interest when the gap length s is reduced below some value which is a function of applied pulse length, longitudinal E field and possibly current density. Such a rapid decrease in the impedance is presumably associated with cathode plasma growth across the gap leading to large percentage changes in s for small gaps. For voltages in the 1 - 2 MV region and pulse lengths up to 100 nsecs. or so this gap length lies in the region 0.5 - 1.0 cms.

These three criteria serve to define a gap configuration for given voltage and impedance. For example, consider the requirements:

$V = 1.5$ MV

$Z \sim 3\ \Omega$ with $k \sim 70$

which mean that $\frac{\nu}{\gamma} \sim 8$.

To avoid pinching we must make $r_c/s > 16$. The Child-Langmuir relation gives

$r_e/s = \sqrt{\frac{k}{ZV^{1/2}}} = 4.3$ in this case.

Taking s ~ 1.0 cm. to avoid large -dZ/dt makes

$r_c \sim 16$ cm. and $r_e \sim 4.3$ cm.

To distribute the emission area reasonably uniformly over the cathode surface we can use n discreet emission 'patches', each of radius r_e'. Then $n r_e'^2 = r_e^2$. In this case for example we may have 36 emitters, each of radius ~7.5 mm., distributed over the 16 cm. radius cathode surface.

Comments

(i) As mentioned previously, k is not independent of time but always decreases to a greater or less extent during the applied voltage pulse. This means that gaps which only just satisfy the pinch criterion at the beginning or in the middle of the pulse may well not do so towards the end.

(ii) In summary, the pinch criterion suffers from all the usual approximations which make it anything but exact. Non-uniform current density, fringing fields, non-uniform B etc. mean that 20% or so accuracy is the best that one can hope for. In the real case however this is sufficient, particularly as undesirably large radial electron energies will be produced at some ν/γ less than but close to the critical value.